Stabilizing CMFD with Linear Prolongation: lpCMFD

Dean Wang, Sicong Xiao (University of Massachusetts Lowell)
Yulin Xu, Thomas Downar (University of Michigan)
Emily Shemon (Argonne National Laboratory)
Yulong Xing (Ohio State University)

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Outline

• CMFD
• Current stabilization methods
• IpCMFD
• Summary and Remarks
CMFD

• Very effective to accelerate neutron transport iteration, **but**
• Degrades and even fails when the problem thickness becomes large.
• **Stabilization needed.**
Stabilization Techniques

- Multiple transport sweeps
- Underrelaxation
  - $\hat{D}$ underrelaxation: $\hat{D}^{l+1/2} = \theta \hat{D}^{l+1/2} + (1 - \theta) \hat{D}^{l-1/2}$
  - Flux update with underrelaxation: $\phi^{l+1} = \phi^{l+1/2} \left[1 + \theta \left(\frac{\phi^{l+1}_{CMFD}}{\phi^{l+1/2}} - 1\right)\right]
- Artificial Diffusion: $D = \frac{1}{3\Sigma_t} + \theta \Delta$
  - pCMFD (Cho et al., 2003): It is algebraically “equivalent” to $\theta = \frac{1}{4}$
  - odCMFD (Larsen, 2003; Zhu et al., 2016): $\theta = \theta(\Sigma_t \Delta)$
IpCMFD

1D:

\[ \delta \phi_i(x) = \delta \Phi_{i-1/2} + \frac{x - x_{i-1/2}}{x_{i+1/2} - x_{i-1/2}} (\delta \Phi_{i+1/2} - \delta \Phi_{i-1/2}) \]

\[ \delta \Phi_{BC} = \begin{cases} 
\delta \Phi_{i-1/2} = \frac{1}{2} \left[ \left( \Phi_{i-1}^{t+1} - \Phi_{i-1/2}^{t+1/2} \right) + \left( \Phi_{i}^{t+1} - \Phi_{i-1/2}^{t+1/2} \right) \right] \\
\delta \Phi_{i+1/2} = \frac{1}{2} \left[ \left( \Phi_{i+1}^{t+1} - \Phi_{i+1/2}^{t+1/2} \right) + \left( \Phi_{i+1}^{t+1} - \Phi_{i+1/2}^{t+1/2} \right) \right] 
\end{cases} \]

2D:
Comparison of Flux Correction
Fourier Analysis

C=0.6

C=0.8

C=0.9

C=0.99
# 1D Iron-Water Test Problem

<table>
<thead>
<tr>
<th>Region 1</th>
<th>Region 2</th>
<th>Region 3</th>
<th>Region 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Water</strong></td>
<td><strong>Water</strong></td>
<td><strong>Iron</strong></td>
<td><strong>Water</strong></td>
</tr>
<tr>
<td>(Q = 1.0)</td>
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<tr>
<td>(\Sigma_t = 3.3333)</td>
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<td>(\Sigma_s = 3.3136)</td>
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<td>(\Sigma_s = 1.1077)</td>
<td>(\Sigma_s = 3.3136)</td>
</tr>
</tbody>
</table>

Transport: \(S_{10}\) Gauss-Legendre, DD
Fine Mesh: 0.1 cm
Coarse Mesh: 1 cm

![Graph 1](image1)

![Graph 2](image2)
2D Fixed-Source Problem
(Wang and Xiao 2018)

\[ \Sigma_t = 3.0 \text{ cm}^{-1}, c = 0.99 \]
\[ \Sigma_t = 10.0 \text{ cm}^{-1}, c = 0.1 \]

Transport: $S_{12}$ Gauss-Legendre, DD
Fine Mesh: 0.1 cm
Coarse Mesh: 1 cm
2D K-Eigenvalue Problem
(Wang and Xiao 2018)

\[ \Sigma_s = 9.0 \text{ cm}^{-1}, \Sigma_t = 10.0 \text{ cm}^{-1}, \nu \Sigma_f = 1.0 \text{ cm}^{-1} \]
\[ \Sigma_s = 8.1 \text{ cm}^{-1}, \Sigma_t = 9.0 \text{ cm}^{-1}, \nu \Sigma_f = 0.99 \text{ cm}^{-1} \]

Transport: \( S_{12} \) Gauss-Legendre, DD
Fine Mesh: 0.1 cm
Coarse Mesh: 1 cm
An Extension: LR-NDA
(Wang 2016; Xiao, 2017, 2018)

Local Refinement BVP:

\[ \nabla \cdot \left( -\frac{1}{3\Sigma_t} \nabla + \hat{D}_{FM}^{l+1/2} \right) \phi_{local}^{l+1} + (\Sigma_t - \Sigma_s) \phi_{local}^{l+1} = Q \]

BCs:

\[ \phi_{BC}^{l+1} = \frac{1}{2} \left( \frac{\phi^{l+1} \Big{|}_+ + \phi^{l+1} \Big{|}_-}{\phi^{l+1/2} \Big{|}_+ + \phi^{l+1/2} \Big{|}_-} \right) \phi_{L,R}^{l+1/2} \]
LR-NDA

Numerical results – local adaptivity

Transport Sweep #

\[ \Sigma_s = 0.9, \Sigma_t = 1.0, \nu \Sigma_f = 0.1 \]
\[ \Sigma_s = 9.0, \Sigma_t = 9.5, \nu \Sigma_f = 1.0 \]
\[ \Sigma_s = 45.0, \Sigma_t = 50.1, \nu \Sigma_f = 5.0 \]
\[ \Sigma_s = 18.0, \Sigma_t = 19.3, \nu \Sigma_f = 2.0 \]

2D K-eigenvalue Problem
S12 Solution Accelerated with LR-NDA

Normalized Scalar Flux

Normalized K_eff Relative Error

Transport Sweep #
Summary and Remarks

• The new lpCMFD scheme employs a linear prolongation for flux update to replace the standard flux ratio based approach.

• lpCMFD is a stable and effective scheme, which performs better than CMFD and other stabilization techniques.

• It can be easily implemented in any codes with CMFD.

• LR-NDA can be viewed as an extension of lpCMFD since it solves a local refinement BVP to obtain a finer flux than linear interpolation.
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References


Thank You!

More info: http://faculty.uml.edu/Dean_Wang